

Mathematics

Overview

An extended essay in mathematics provides students with an opportunity to demonstrate an appreciation of any aspect of the subject, whether it is:

- the applicability of mathematics to solve both real and abstract problems
- the beauty of mathematics as in, for instance, geometry or fractal theory
- the elegance of mathematics in the proving of theorems as in, for example, number theory
- the origin and subsequent development of a branch of mathematics over a period of time, measured in tens, hundreds or thousands of years
- the links between different branches of mathematics and the powerful structures that enable many seemingly different problems to be solved by a single theory
- the way that a branch of mathematics has been born, or has flourished, as a result of technology.

These are just some of the many different ways that mathematics can be enjoyable or useful, or, as in many cases, both.

Choice of topic

The extended essay may be written on any topic that has a mathematical focus and it need not be confined to the theory of mathematics itself.

Students may choose mathematical topics from fields such as engineering, the sciences or the social sciences, as well as from mathematics itself. Statistical analyses of experimental results taken from other subject areas are also acceptable, provided that they focus on the modelling process and discuss the limitations of the results; such essays should not include extensive non-mathematical detail. A topic selected from the history of mathematics may also be appropriate, provided that a clear line of mathematical development is demonstrated. Concentration on the lives of, or personal rivalries between, mathematicians would be irrelevant and would not score highly on the assessment criteria.

It should be noted that the assessment criteria give credit for the nature of the investigation and for the extent that reasoned arguments are applied to an appropriate research question. Students should avoid choosing a topic that gives rise to a trivial research question or one that is not sufficiently focused to allow appropriate treatment within a suitably sized essay. Students will normally be expected either to extend their knowledge beyond that encountered in the Diploma Programme mathematics course they are studying, or to apply techniques used in their mathematics course to modelling in an appropriately chosen topic. However, it is very important to remember that it is an essay that is being written, not a research paper for a journal of advanced mathematics, and no result, however impressive, should be quoted without evidence of the student's real understanding of it.

The following examples of titles for mathematics extended essays are intended as guidance only. The pairings illustrate that focused topics (indicated by the first title) should be encouraged rather than broad topics (indicated by the second title).

- “Prime numbers in cryptography” **is better than** “Prime numbers”.
- “The Hausdorff dimension of fractal sets” **is better than** “Fractals”.
- “Continued fractions in birth–death processes” **is better than** “Continued fractions”.

- “The proof of the law of quadratic reciprocity” **is better than** “CF Gauss: the mathematician”.
- “Using graph theory to minimize cost” **is better than** “Graph theory”.

The next examples illustrate ways in which mathematics topics can be refined and successfully approached as extended essays.

Topic	The geometry of navigation
Research question	What was the role of mathematics, and geometry in particular, in navigation when we relied on the stars? Does it still play a part now we have man-made satellites?
Approach	Using one of the two geometric representations of the earth (spherical or ellipsoidal), describe how maps and charts were produced to assist navigators in the past.

Topic	Square–triangular numbers and Pell’s equation
Research question	How many square numbers are also triangular numbers, where are they, and what other problems lead to Pell’s equation?
Approach	A description of square and triangular numbers, and how the locations of numbers that are both are solutions of Pell’s equation. Some other problems, perhaps in number theory and geometry, that lead to the equation could be described, with a brief history of the equation included.

Topic	The exponential function and the measurement of age and growth
Research question	How does the exponential function, and its calculus, inform areas of science such as nuclear physics, geology, anthropology or demography?
Approach	Use one of the settings where exponential growth applies, perhaps modelling the world’s population, to describe the phenomena. Show how it is applicable in mathematical models of other real situations.

Topic	Approximation of irrational numbers by rational numbers
Research question	How well can π , e , $\sqrt{2}$ and other irrationals be approximated by rational numbers?
Approach	Use the decimal representation of irrational numbers as a starting point to introduce approximation by rationals. Show how a continued fraction expansion of an irrational can also provide rational approximation, and discuss error bounds and orders of approximation.

Topic	Archimedes’ calculations of areas
Research question	What is the legacy of Archimedes’ calculations of circular and parabolic areas in today’s methods of integration?
Approach	Describe how Archimedes determined the area of a circle by using inscribed polygons, leading also to his measurement of π . Continue with a description of his method of discovery for calculating the area of a parabola.

Treatment of the topic

Whatever the title of the extended essay, students must apply good mathematical practice that is relevant to the chosen topic. Data must be analysed using appropriate techniques; arguments correctly reasoned; situations modelled using correct methodology; problems clearly stated and techniques at the correct level of sophistication applied to their solution. There must be sufficient explanation and commentary throughout the extended essay to ensure that the reader does not lose sight of the purpose of the essay in a mass of mathematical symbols, formulae and analysis.

The unique disciplines of mathematics must be respected throughout. Relevant graphs and diagrams are often important and should be incorporated in the body of the essay, not relegated to an appendix. However, lengthy printouts, tables of results and computer programs should not be allowed to interrupt the development of the essay, and should appear separately as footnotes or in an appendix. Proofs of key results may be included but proofs of standard results should be either omitted or, if they illustrate an important point, included in an appendix.

Frequent reference to the assessment criteria by both the supervisor and the student will help keep a sharper focus on the project.

Interpreting the assessment criteria

Criterion A: research question

Although the aim of the essay can best be defined in the form of a question, it may also be presented as a statement or proposition for discussion.

Achievement level	Descriptor
0	The research question is not stated in the introduction or does not lend itself to a systematic investigation in an extended essay in the subject in which it is registered.
1	The research question is stated in the introduction but is not clearly expressed or is too broad in scope to be treated effectively within the word limit.
2	The research question is clearly stated in the introduction and sharply focused, making effective treatment possible within the word limit.

Criterion B: introduction

The introduction should relate the research question to existing subject knowledge: the student's personal experience or particular opinion is rarely relevant here.

Achievement level	Descriptor
0	Little or no attempt is made to set the research question into context. There is little or no attempt to explain the significance of the topic.
1	Some attempt is made to set the research question into context. There is some attempt to explain the significance of the topic and why it is worthy of investigation.
2	The context of the research question is clearly demonstrated. The introduction clearly explains the significance of the topic and why it is worthy of investigation.

Criterion C: investigation

The number of sources that require consultation will be dependent on the research question that the essay addresses. It must be sufficient, but not unnecessarily long, and the various items must, after being consulted, contribute to the essay in a meaningful way.

Achievement level	Descriptor
0	There is little or no evidence that sources have been consulted or data gathered, and little or no evidence of planning in the investigation.
1	A range of inappropriate sources has been consulted, or inappropriate data has been gathered, and there is little evidence that the investigation has been planned.
2	A limited range of appropriate sources has been consulted, or data has been gathered, and some relevant material has been selected. There is evidence of some planning in the investigation.
3	A sufficient range of appropriate sources has been consulted, or data has been gathered, and relevant material has been selected. The investigation has been satisfactorily planned.
4	An imaginative range of appropriate sources has been consulted, or data has been gathered, and relevant material has been carefully selected. The investigation has been well planned.

Criterion D: knowledge and understanding of the topic studied

Clear evidence of understanding of the chosen content of the essay is more important than any attempt to exhibit an unnecessarily wide mathematical knowledge. The level of knowledge displayed should reflect the mathematics acquired in the classroom and any additional information that the research for the essay has provided.

Achievement level	Descriptor
0	The essay demonstrates no real knowledge or understanding of the topic studied.
1	The essay demonstrates some knowledge but little understanding of the topic studied. The essay shows little awareness of an academic context for the investigation.
2	The essay demonstrates an adequate knowledge and some understanding of the topic studied. The essay shows some awareness of an academic context for the investigation.
3	The essay demonstrates a good knowledge and understanding of the topic studied. Where appropriate, the essay successfully outlines an academic context for the investigation.
4	The essay demonstrates a very good knowledge and understanding of the topic studied. Where appropriate, the essay clearly and precisely locates the investigation in an academic context.

Criterion E: reasoned argument

In a mathematics extended essay, the terms “reasoning” and “argument” can apply to the essay overall and, with a somewhat different interpretation, to one or more sub-sections of the essay, such as in the proof of a theorem. The mark awarded should reflect both of these aspects.

Achievement level	Descriptor
0	There is no attempt to develop a reasoned argument in relation to the research question.
1	There is a limited or superficial attempt to present ideas in a logical and coherent manner, and to develop a reasoned argument in relation to the research question.
2	There is some attempt to present ideas in a logical and coherent manner, and to develop a reasoned argument in relation to the research question, but this is only partially successful.
3	Ideas are presented in a logical and coherent manner, and a reasoned argument is developed in relation to the research question, but with some weaknesses.
4	Ideas are presented clearly and in a logical and coherent manner. The essay succeeds in developing a reasoned and convincing argument in relation to the research question.

Criterion F: application of analytical and evaluative skills appropriate to the subject

The words “appropriate to the subject” are paramount here, given the breadth of mathematics today. Essays based on mathematical proof require skills in deductive reasoning and structuring; essays of an investigative nature, answering a research question or hypothesis, involve interpretative skills; while mathematical modelling requires an ability to formulate correctly a problem in mathematical terms.

Achievement level	Descriptor
0	The essay shows no application of appropriate analytical and evaluative skills.
1	The essay shows little application of appropriate analytical and evaluative skills.
2	The essay shows some application of appropriate analytical and evaluative skills, which may be only partially effective.
3	The essay shows sound application of appropriate analytical and evaluative skills.
4	The essay shows effective and sophisticated application of appropriate analytical and evaluative skills.

Criterion G: use of language appropriate to the subject

The purpose of language in mathematics is, as in other subjects, to communicate. In an extended essay, it should communicate clearly, not just to an examiner who is likely to be more mathematically sophisticated than the student, but also to an interested reader with a similar mathematical competence to the student. Mathematical “name-dropping” should be avoided.

Achievement level	Descriptor
0	The language used is inaccurate and unclear. There is no effective use of terminology

	appropriate to the subject.
1	The language used sometimes communicates clearly but does not do so consistently. The use of terminology appropriate to the subject is only partly accurate.
2	The language used for the most part communicates clearly. The use of terminology appropriate to the subject is usually accurate.
3	The language used communicates clearly. The use of terminology appropriate to the subject is accurate, although there may be occasional lapses.
4	The language used communicates clearly and precisely. Terminology appropriate to the subject is used accurately, with skill and understanding.

Criterion H: conclusion

“Consistent” is the key word here: the conclusion should develop out of the argument and not introduce new or extraneous matter. It should not repeat the material of the introduction; rather, it should present a new synthesis in light of the discussion.

Achievement level	Descriptor
0	Little or no attempt is made to provide a conclusion that is relevant to the research question.
1	A conclusion is attempted that is relevant to the research question but may not be entirely consistent with the evidence presented in the essay.
2	An effective conclusion is clearly stated; it is relevant to the research question and consistent with the evidence presented in the essay. It should include unresolved questions where appropriate to the subject concerned.

Criterion I: formal presentation

This criterion relates to the extent to which the essay conforms to academic standards about the way in which research papers should be presented. The presentation of essays that omit a bibliography or that do not give references for quotations is deemed unacceptable (level 0). Essays that omit one of the required elements—title page, table of contents, page numbers—are deemed no better than satisfactory (maximum level 2), while essays that omit two of them are deemed poor at best (maximum level 1).

Word count is rarely an important factor in a good mathematics extended essay. Since equations and formulae (indicating the student’s mathematical reasoning) are not included in the word count, a substantial essay can be produced that contains comparatively few words. Concise, elegant mathematics supported by graphs, diagrams and important proofs that do not interrupt the development of the essay are encouraged. However, an essay that is excessive in length will be penalized, especially if this is because of unnecessary content. There is no mandatory minimum length for an essay in mathematics, and credit will be given for organizing the content in an efficient readable style, rather than for a page or word count. Mastery of appropriate concepts, and an ability to present these in an effective way using mathematical means, should be the aim.

Achievement level	Descriptor
0	The formal presentation is unacceptable, or the essay exceeds 4,000 words.
1	The formal presentation is poor.
2	The formal presentation is satisfactory.
3	The formal presentation is good.
4	The formal presentation is excellent.

Criterion J: abstract

The abstract is judged on the clarity with which it presents an overview of the research and the essay, not on the quality of the research question itself, nor on the quality of the argument or the conclusions.

Achievement level	Descriptor
0	The abstract exceeds 300 words or one or more of the required elements of an abstract (listed above) is missing.
1	The abstract contains the elements listed above but they are not all clearly stated.
2	The abstract clearly states all the elements listed above.

Criterion K: holistic judgment

Qualities that are rewarded under this criterion include intellectual initiative, insight and depth of understanding, and creativity. Such qualities can be displayed in different ways, depending on the type of mathematics extended essay.

Achievement level	Descriptor
0	The essay shows no evidence of such qualities.
1	The essay shows little evidence of such qualities.
2	The essay shows some evidence of such qualities.
3	The essay shows clear evidence of such qualities.
4	The essay shows considerable evidence of such qualities.